Experiments on granular flow in a cylindrical couette

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Overview

• What are granular materials?
  – Dissipative interactions
  – Negligible thermal fluctuations ($K_B T << mgd$)

• Size ranges:

<table>
<thead>
<tr>
<th>Size Range (m)</th>
<th>Example of material</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-6} &lt; d &lt; 10^{-3}$</td>
<td>Powders, fine particles</td>
</tr>
<tr>
<td>$10^{-3} &lt; d &lt; 0.1$</td>
<td>Grains, cereals, sand, salt</td>
</tr>
<tr>
<td>$d &gt; 0.1$</td>
<td>Pebbles, rocks, boulders</td>
</tr>
</tbody>
</table>
Overview

• Flow of granular material characterized by different regimes

• No single constitutive equation valid over all regimes. No unified theory exists

• To quantify regime. \[ R = \frac{\rho d_p^2 \cdot \gamma^2}{N_t} \]
Flow regimes

Rapid flow regime: collisional contact

Intermediate flow regime: collisional and frictional contact

Slow flow regime: slow deformation, frictional contact

Image from Forterre & Pouliquen (2008)
When does flow occur?

• Yield Condition
  – For a block on a plane, static friction
    \[ F_t \leq \mu_s W + C \]
  – Block of granular material relative to solid surface
    \[ |T_w| \leq N_w \tan \delta' + c_w \]

\( \delta \text{ is angle of wall friction} \)
Yield condition

• Rupture along a layer segregating granular material

\[ |T_r| \leq N_r \tan \phi_* + c \]

\( \phi_* \) is angle of internal friction

• General condition: When direction of rupture layer not known

\[ F(I_1, I_2, \nu) = 0 \]

\( I_1, I_2 \) are invariants of stress tensor
Modelling flow regimes

• Slow flow
  – Plasticity models from soil mechanics
  – Incorporates Yield condition, Flow rules, Plastic deformations and dissipations

• Rapid flow
  – Kinetic theory based model
  – binary collisions of spherical particles, dissipation
Some features of dense granular flow

- A constitutive model of dense flow should
  - Predict zones of flow and no-flow
  - Predict extent of shear layer
  - Exhibit stress independent of rate of deformation
  - Valid for a variety of flow configurations & boundary conditions
  - Include effects of shape and size of granular material
  - Ex: Cosserat model, friction-collisonal model
Features studied in experiment

- Imaging of rice particles in cylindrical couette
  - Velocity profiles in shear layer
  - Radial, tangential & angular velocity distributions
  - Spatial & time correlations of velocity fluctuations
Develop theories of granular flow

– Correlations of velocity in time & space (ie, assumptions of molecular chaos and binary collisions are violated)

– Type of velocity distribution, universality for different boundary conditions, particle types, etc

– Ordering & clustering of particles
Experimental Apparatus

1. Kodak Megaplus ES310 camera with Macrozoom lens

2. Inner cylinder diameter 35.5 mm, Outer cylinder diameter 70 mm

3. 12V DC motor by RS components with gear box

4. Granular material -> Rice particle coated for better reflection

5. Radial translational stand

6. Inlet for airflow
Experimental Technique

• Porous plate with inlet for air for distribution
• Inner cylinder rotated at 2 rpm
• 50 seconds video taken at 60 fps
• Ten movies taken for each flow rate of air
• Experiment repeated for 5 different aeration rates (Loose Random Packing)
Captured images at 60fps
XCAP software, Gain 4x

Thresholded images
Maximum threshold to prevent overlapping

Fitted images
ImageJ fits ellipses and gives centroid, axis, angle, area, circularity
Particle search algorithm

- $r_{i,j}$ is position of particle $j$ in frame $i$
- $r_{i+1,k}$ is position of particle $k$ in frame $i+1$
- $d_s$ is the search radius
- For a given frame $i$, the value $|r_{i+1,k} - r_{i,j}|$ is calculated $\forall k,j$
- If $|r_{i-1,k} - r_{i,j}| < d_s$, then particle is detected
- If more than 1 particle detected, particle is discarded
- If there is no particle detected, particle is deemed lost

- From displacements, we get vertical and horizontal velocities for detected particles in each frame
Finding velocities

\[ v_r = |v| \cos(\phi - \theta) \]
\[ v_\theta = |v| \sin(\phi - \theta) \]
\[ \alpha = \pi + \theta - \chi \]
\[ \omega = \frac{d\alpha}{dt} \]
Binning Particles

• Particles radially binned

• Bin centers 1 mm apart, bin width 2 mm

• Average property assigned to center of bin

• Sliding average for smoother profiles
Average properties

• If $\psi$ is some property of the particle, the average value $\langle \psi \rangle$ is defined as

$$\langle \psi \rangle = \frac{\sum \psi_i}{N}$$

• For large $N$, assumed that this mean is same as property calculated by kinetic theory
Mean tangential velocity

The profile shows exponential decay with 5-6 minor axis lengths.
Mean radial velocity

Radial velocities are approximately order of magnitude smaller
Mean angular velocity

\[
\frac{\omega(R_2 - R_1)}{R_1 \omega_w}
\]

Angular velocity profiles

\[
\frac{r}{R_2 - R_1}
\]
Vorticity is defined as, for flow in the couette, this is
\[ (\nabla \times \nu) \]
Calculated by fitting smooth 4th order splines

Difference between the angular and half the vorticity is evidence of stress asymmetry (Ananda, Patra and Nott, 2008)
Correlations

- The fluctuation velocity can be found from overall mean as $c = (v - \bar{v}) / \sigma$ in each bin.

- Spatial correlation:

$$f(\Delta x) = \frac{1}{N} < c(x)c(x + \Delta x) >$$

- Time correlation:

$$f(\tau) = \frac{1}{N} < c(t)c(t + \tau) >$$
Time correlation

Tangential fluctuations

Radial fluctuations

$f(\tau)$ vs $t\gamma$ for different distances (0mm, 60mm, 120mm, 180mm, 240mm).
Spatial correlation

Tangential fluctuations decay within roughly one major axis length.

Radial fluctuations show solid-like correlation over long range, and this is verified by visual observation.
Velocity distributions

• For a molecular fluid with elastic collisions, the velocity distribution is gaussian

• Non-gaussian distributions -> dissipation due to collision and friction

• Help in developing and verifying constitutive equations
Tangential velocity distributions

- Area under distribution normalized to unity
- The tail is not exponential
- Deviates from gaussian for

\[ f \left( \frac{v_\theta - v_\theta}{\sigma_{\text{tot}}} \right) \]
Tangential velocity distributions

- Power law distribution up to moderately large fluctuations
- More statistics with higher fps may be required
Radial velocity distribution

\[ f\left(\frac{v_r - \bar{v}_r}{\sigma_{tot}}\right) \]

\[ \log_{10}\left(\frac{v_r - \bar{v}_r}{\sigma_{tot}}\right) \]

\[ v_r - \bar{v}_r \]

\[ \sigma_{tot} \]

\[ \log_{10}\left(\frac{v_r - \bar{v}_r}{\sigma_{tot}}\right) \]

\[ 0mm \]

\[ 60mm \]

\[ 120mm \]

\[ 180mm \]

\[ 240mm \]
Radial velocity distributions

\[ f \left( \frac{v_r - \bar{v}_r}{\sigma_{tot}} \right) \]

\[ \frac{v_r - \bar{v}_r}{\sigma_{tot}} \]
Summary

- The mean velocity profiles were obtained. The shear layer matches with that predicted by continuum models for slow flow.

- Evidence of cosserat type effects was observed.

- Spatial correlations of velocity fluctuations exists, however distinct from reported correlation for gravity driven flow.

- The velocity distribution is non-gaussian, and obeys a power-law behavior for larger velocities.

- Does phenomena analogous to convection rolls occur?

- Liquid-crystal like ordering of particles was observed.
Outlook & Future experiments

• Will a higher frame-rates uniquely identify the type of velocity distribution observed

• Are similar statistics obtained for oblate ellipsoids, and for different flow parameters?

• Does the angular velocity profile match with that predicted by cosserat model? The stress profiles at the outer surface.

• Can the ordering of particles under flow be characterized with a angular distribution function?
References


• K.S Ananda, J. Patra & P.R Nott “Experimental evidence that dense granular materials are Cosserat (micropolar) continua”, (TBP)